Dynamic Stability of a Cantilevered Timoshenko Beam on Partial Elastic Foundations Subjected to a Follower Force

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This paper presents the dynamic stability of a cantilevered Timoshenko beam with a concentrated mass, partially attached to elastic foundations, and subjected to a follower force. Governing equations are derived from the extended Hamilton's principle, and FEM is applied to solve the discretized equation. The influence of some parameters such as the elastic foundation parameter, the positions of partial elastic foundations, shear deformations, the rotary inertia of the beam, and the mass and the rotary inertia of the concentrated mass on the critical flutter load is investigated. Finally, the optimal attachment ratio of partial elastic foundation that maximizes the critical flutter load is presented.

Key Words : Partial Elastic Foundations, Dynamic Stability, Follower Force, Cantilevered Timoshenko Beam

1. Introduction

The dynamic stability of a beam on elastic foundations subjected to nonconservative forces has been an interesting subject to many researchers in engineering areas like mechanical and aerospace engineering. For example, a pipe system conveying fluid on the ground or a rocket or missile with thrust can be treated as a structure subjected to a follower force which is nonconservative.

This type of study was initiated by Smith and Herrmann (1972). They investigated the critical follower load of Beck's column supported entirely by an elastic foundation with various stiffness. They reached an interesting result that the critical load for flutter remains unchanged regardless of both the existence and the stiffness of the foundation. Sundararajan (1974) extended Smith and Herrmann's work by investigating the stability of columns on elastic foundations under conservative and nonconservative forces when the elastic foundation modulus has a similar distribution to that of the mass of the column. He also concluded that the critical load is independent of elastic foundations.

Later, Anderson (1975) studied the stability of

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a uniform viscoelastic cantilever resting on an elastic foundation, carrying a tip mass, and subjected to a follower force. He examined the effects of the rotary inertia of the beam, the size and rotary inertia of the tip mass, and the foundation modulus on the value of the critical flutter load. In this study, he found that the rotary inertias of both the beam and the tip mass affect somewhat importantly the critical follower load of the beam on an elastic foundation.

Hauger and Vetter (1976) extended the study on the dynamic stability of a cantilevered Euler-Bernoulli (Sundararajan, 1976) column on an elastic foundation under a follower force by employing different shapes of foundation modulus distribution such as the uniform, linear, and parabolic ones. Jacoby and Elishakoff (1986) rechecked Anderson's study on the dynamic stability of a cantilever beam on an elastic foundation, carrying a tip mass, the so-called Pfluger's column, under a follower force at its free end.

In parallel with these studies, the stability of a column with different supporting conditions subjected to a follower force has also been studied. Sundararajan (1976) investigated the vibration and stability of a beam with an elastic end support by varying support stiffness.

Elishakoff and Wang (1987) studied the dynamic stability of a cantilevered Euler-Bernoulli column partially attached to an elastic foundation subject to a follower force. They investigated the effect of the attachment ratio, which is the ratio of the length supported by an elastic foundation to the entire length of the column, on the critical flutter load. They presented how the ratio of the critical flutter load for the elastic foundation stiffness $K=20\sim100$ to that for $K=0$ varies with the value of the attachment ratio. According to their results, the critical flutter load initially increases up to the maximum as the attachment ratio increases to around 0.7 and then decreases until the Smith-Hermamn case is recovered at total attachment. It is, however, believed that they made some mistakes in their calculation. One of the purposes of this paper is to correct their results.

Chen and Ku (1992) employed the finite element method to investigate the influence of the rotary inertia, the size of a concentrated mass, and the elastic foundation parameter on the critical load of the cantilevered Timoshenko beam supported entirely by elastic foundations, carrying a tip mass, under follower forces.

Recently, Maurizi and Bambill (2002) recommended that Langthejem and Sugiyama (2000) on the dynamic stability of columns subject to follower loads should include the studies on the stability of columns supported entirely or partially by elastic foundations subjected to follower loads. Therefore, it is strongly desired to study the dynamic stability of a Timoshenko beam partially supported by elastic foundation under a follower force. The purpose of this paper is to investigate the effect of the rotary inertia, the shear deformation, the size and the rotary inertia of the concentrated mass, and the elastic foundation stiffness on the dynamic stability of a Timoshenko beam partially resting on elastic foundations, under a follower force.

2. Theory

2.1 Mathematical model and equation of motion

Figure 1 shows the mathematical model of a cantilevered beam partially supported by the elastic foundation of stiffness k, carrying a concentrated mass M, under a follower force P. In order

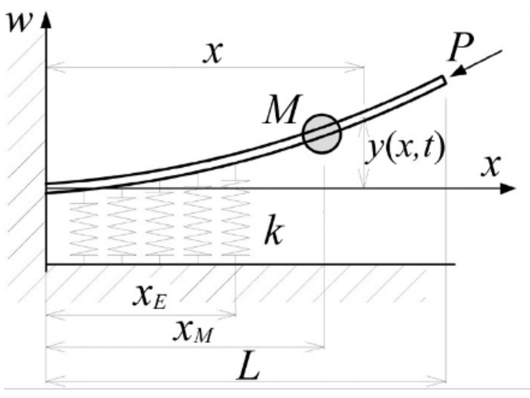

Fig. 1 Mathematical model of a cantilevered beam on partial elastic foundations subjected to a follower force

to drive the governing equation of motion for the model shown in Fig. 1, the energy equations can be expressed as

$$T=\frac{1}{2}\int_0^L \rho A y_t^2\,dx+\frac{1}{2}\int_0^L \rho I \phi_t^2\,dx+\frac{1}{2}My_t^2(x_M,t)+\frac{1}{2}J\phi_t^2(x_M,t) \tag{1}$$

$$V=\frac{1}{2}\int_0^L EI\phi_x^2\,dx+\frac{1}{2}\int_0^L k'AG(y_x-\phi)^2\,dx+\frac{1}{2}\int_0^{x_E} ky^2\,dx \tag{2}$$

$$W_c=\frac{1}{2}\int_0^L Py_x^2\,dx \tag{3}$$

$$\delta W_{nc}=-P\phi(L,t)\,\delta y(L,t) \tag{4}$$

where, T is the kinetic energy, V is the elastic potential energy from both beam and foundation, W_c is the conservative work done by the follower force, δW_{nc} is the virtual work by the follower force, ρ is the density of the beam, y is the transverse displacement, ϕ is the bending slope, x_M is the position of the concentrated mass, x_E is the end position of the partial elastic foundation, k' is the shear modulus of a beam, EI is the flexural rigidity of a beam, L is the length of a beam, J is the rotary inertia of the concentrated mass, A is the cross sectional area, and the subscripts x and t represent differentiation with respect to the axial coordinate and time, respectively.

Substituting Eqs. (1)-(4) into the extended Hamilton principle

$$\delta\int_{t_1}^{t_2}(T-V+W_c)\,dt+\int_{t_1}^{t_2}\delta W_{nc}\,dt=0 \tag{5}$$

gives

$$\begin{aligned}&\int_{t_1}^{t_2}\Big[\int_0^L(\rho A y_{tt}\delta y+\rho I\phi_{tt}\delta\phi+EI\phi_x\delta\phi_x\\&-Py_x\delta y_x+k'AG(y_x-\phi)\,\delta(y_x-\phi))\,dx\\&+\int_0^{x_E}ky\delta y\,dx+My_{tt}(x_M,t)\,\delta y(x_M,t)\\&+J\phi_{tt}(x_M,t)\,\delta\phi(x_M,t)\Big]dt\\&+\int_{t_1}^{t_2}\Big\{P\phi(L,t)\,\delta y(L,t)\Big\}\,dt=0\end{aligned} \tag{6}$$

2.2 Application of finite element method

The finite element method is applied to obtain the discretized equation of motion from Eq. (6). The beam is divided into N elements of an equal length l, and the following nondimensional coordinates and local coordinates are introduced:

$$\begin{aligned}&x'=x-(i-1)\,l,\; x'_M=x_M-(N-1)\,l,\; \xi_M=\frac{x'_M}{l}\\&x'_E=x_E-(d-1)\,l,\; \xi=\frac{x'}{l},\; \xi_s=\frac{x'_E}{l},\; \eta=\frac{y}{l}\end{aligned} \tag{7}$$

Separating variables by using Eq. (7), the discretized equation of Eq. (6) can be expressed as

$$\begin{aligned}&\sum_{i=1}^{N}\Big[\int_0^1\Big\{\rho A l^3\eta_{tt}^{(i)}\delta\eta^{(i)}+\rho I l\phi_{tt}^{(i)}\delta\phi^{(i)}+\frac{EI}{l}\phi_\xi^{(i)}\delta\phi_\xi^{(i)}\\&-Pl\eta_\xi^{(i)}\delta\eta_\xi^{(i)}+k'AGl(\eta_\xi^{(i)}-\phi^{(i)})\,\delta(\eta_\xi^{(i)}-\phi^{(i)})\Big\}\,d\xi\Big]\\&+\sum_{i=1}^{d}\Big[\int_0^{\xi_2}(kl^3\eta^{(i)}\delta\eta^{(i)})\,d\xi\Big]\\&+Ml^2\eta_{tt}^{(N)}(\xi_M,t)\,\delta\eta^{(N)}(\xi_M,t)+J\phi_{tt}^{(N)}(\xi_M,t)\,\delta\phi^{(N)}(\xi_M,t)\\&+P\phi(1,t)^{(N)}\delta\eta(1,t)^{(N)}=0\end{aligned} \tag{8}$$

Assuming the solutions in the form of

$$\eta(\xi,t)=\eta(\xi)\,e^{st},\; \phi(\xi,t)=\phi(\xi)\,e^{st} \tag{9}$$

and introducing the following nondimensional parameter,

$$\begin{aligned}&\alpha=\frac{M}{\rho AL},\; \beta=\frac{J}{\rho AL^3},\; \mu=\frac{x_M}{L},\; K=\frac{kL^4}{EI},\; \sigma=\frac{x_E}{L}\\&Q=\frac{PL^2}{EI},\; \lambda^2=\frac{\rho AL^4 s^2}{EI},\; R=\frac{1}{AL^2},\; S=\frac{k'AGL^2}{EI}\end{aligned} \tag{10}$$

Eq. (8) can be finally expressed as

$$\begin{aligned}&\sum_{i=1}^{N}\Big[\int_0^1\Big\{\frac{\lambda^2}{N^4}\eta^{(i)}\delta\eta^{(i)}+\frac{\lambda^2 R}{N^4}\phi^{(i)}\delta\phi^{(i)}\\&+\phi_\xi^{(i)}\delta\phi_\xi^{(i)}-\frac{Q}{N^2}\eta_\xi^{(i)}\delta\eta_\xi^{(i)}\\&+\frac{S}{N^2}(\eta_\xi^{(i)}-\phi^{(i)})\,\delta(\eta_\xi^{(i)}-\phi^{(i)})\Big\}\,d\xi\Big]\\&+\sum_{i=1}^{d}\Big[\int_0^{\xi_s}\Big(\frac{K}{N^4}\eta^{(i)}\delta\eta^{(i)}\Big)\,d\xi\Big]\\&+\frac{\alpha\lambda^2}{N^3}\eta^{(N)}(\xi_M)\,\delta\eta^{(N)}(\xi_M)+\frac{\beta\lambda^2}{N}\phi^{(N)}(\xi_M)\,\delta\phi^{(N)}(\xi_M)\\&+\frac{Q}{N^2}\phi(1)^{(N)}\delta\eta(1)^{(N)}=0\end{aligned} \tag{11}$$

In Eq. (11), α is the ratio of the concentrated mass to the mass of the beam, β is the rotary inertia parameter of the concentrated mass, μ is the dimensionless position of the concentrated mass, K is the elastic foundation parameter, σ is the dimensionless end position of the region supported by the elastic foundation, Q is the dimensionless follower force, λ is the dimensionless natural frequency, R and S are the rotary inertia and the shear deformation of the beam, respectively.

3. Numerical Analyses Results and Discussion

Numerical analyses on the dynamic stability of a cantilevered Timoshenko beam partially attached to an elastic foundation under a follower force are performed by employing the finite element method. The accuracy of numerical results obtained was checked by comparing to the results in Ref.(Jacoby and Elishakoff, 1986) for the case of an Euler-Bernoulli beam supported entirely by an elastic foundation subject to a follower force. The critical follower force of Q/π^2 obtained in the present study is 2.0317 which is only 0.0098% different from the value of 2.0315 from Ref.(Jacoby and Elishakoff, 1986).

3.1 The effect of a partially attached elastic foundation

Figure 2 shows the change of the critical follower force with the elastic foundation parameter for an Euler-Bernoulli beam, a beam considering shear deformations only, a Rayleigh beam, and a Timoshenko beam when the beam is supported entirely by an elastic foundation.

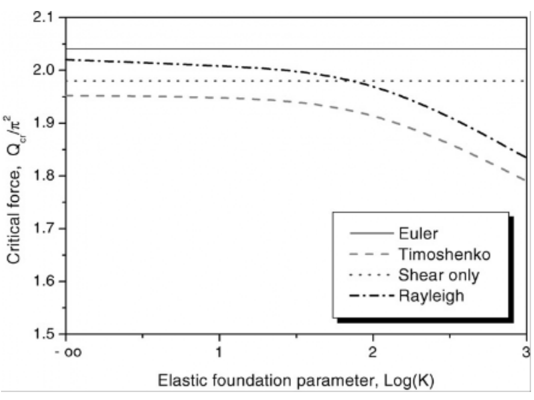

Fig. 2 Critical force for various types of a beam

As shown in the figure, the critical follower force remains constant with the change of the elastic foundation parameter for both an Euler-Bernoulli beam and the beam considering shear deformation only, for a given shear deformation parameter. However, the critical follower force decreases as the elastic foundation parameter increases when the rotary inertia is considered for both a Rayleigh beam and a Timoshenko beam.

Figure 3 shows the critical follower force for the beam supported entirely by the elastic foundation for various shear deformation parameters when the rotary inertia parameter of $R=0.0$. As can be seen in the figure, the shear deformation parameter S has little effect on the critical follower force for $S \geq 10^4$.

Figure 4 presents the effect of the rotary inertia

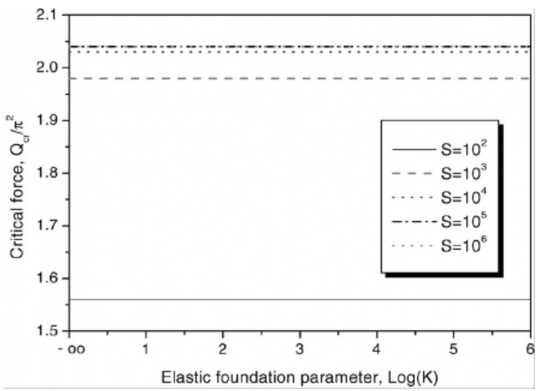

Fig. 3 Critical force depending on the shear deformation parameter, S

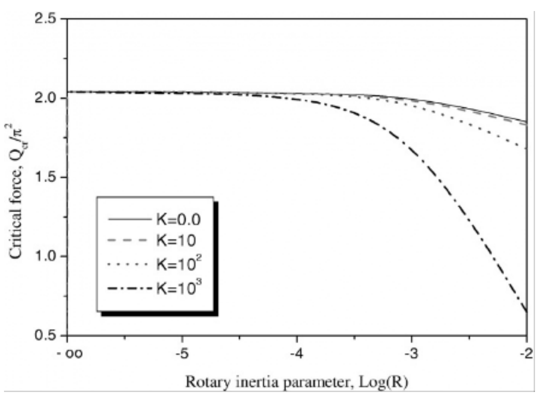

Fig. 4 Critical force depending on the rotary inertia parameter, R

parameter on the critical follower force of a beam supported entirely by the foundation for various elastic foundation parameters when the shear deformation parameter $S = 10^6$. As shown in the figure, the elastic foundation parameter has little effect on the critical follower force for the rotary inertia parameter of $R \leq 10^{-4}$.

Figures 5 and 6 show the change of the critical follower force with the length of the region along which the elastic foundation is attached to the beam for an Euler-Bernoulli beam and a Timoshenko beam, respectively. In both figures, the critical follower force increases at first and decreases later and then increases again as the dimensionless end position of the elastic foundation ξ_s moves from the fixed end to the free end of the beam for a given elastic foundation parameter.

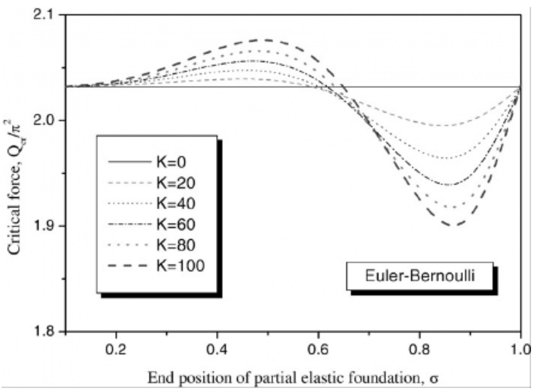

Fig. 5 Critical force depending on the partial elastic foundation parameter (Euler-Bernoulli beam)

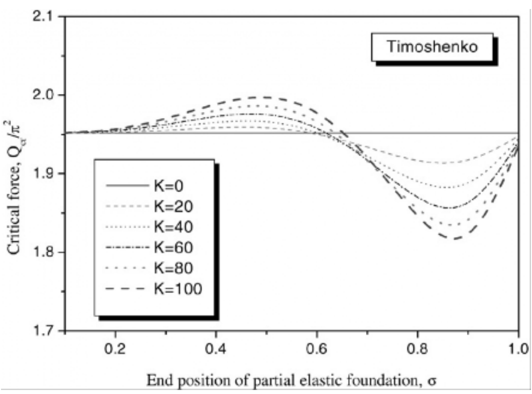

Fig. 6 Critical force depending on the partial elastic foundation parameter (Timoshenko beam)

The critical follower forces for a Timoshenko beam are found to be smaller than those for an Euler-Bernoulli beam. It is also found that the maximum critical follower force occurs at $\xi_s = 0.5$, i.e. when the elastic foundation is attached from the fixed end to the mid point of the beam, for a given elastic foundation parameter.

3.2 Effect of a concentrated mass

Figures 7 and 8 present the mass effect of the tip mass on the critical follower force of a Timoshenko beam supported entirely by the foundation for different elastic foundation parameters without and with including the rotary inertia of the tip mass, respectively.

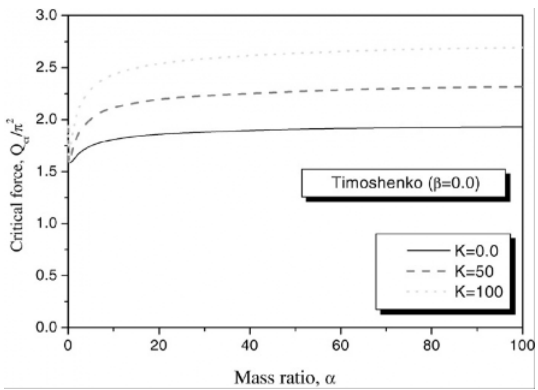

Fig. 7 Critical force depending on the mass ratio without considering the rotary inertia of the tip mass

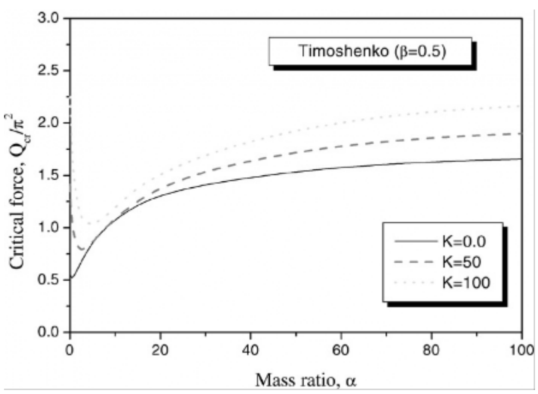

Fig. 8 Critical force depending on the mass ratio taking into account the rotary inertia of the tip mass

As shown in Fig. 7, when only the mass effect of the tip mass is considered, the critical follower force increases in general as the elastic foundation parameter increases for a given mass ratio α. However, the opposite phenomenon is observed for a very small value of $\alpha \leq 0.3$. When both the mass and rotary effects of the tip mass are taken into account, the critical follower force increases as the elastic foundation parameter increases for a given mass ratio α as shown in Fig. 8.

The rotary inertia of the tip mass affect somewhat significantly the critical follower force for a given elastic foundation parameter.

4. Conclusions

The following results were obtained by performing the numerical analyses on the dynamic stability of a Timoshenko beam supported partially by an elastic foundation under a follower force.

(1) The shear deformation of a beam has no effect on the critical follower force as the elastic foundation parameter varies, while the rotary inertia of the beam affects it significantly.

(2) For a given elastic foundation parameter, the critical follower force increases at first and decreases later, and then increases again as the region supported by the elastic foundation expands from the fixed end to the free end of the beam. The maximum critical follower force is obtained when the elastic foundation is attached from the fixed end to the mid point of the beam.

(3) When a tip mass exists, the critical follower force increases in general as the elastic foundation parameter increases for a given mass ratio of α when the mass effect of the tip mass is considered. The opposite phenomenon is, however, observed for a very small value of $\alpha \leq 0.3$. The critical follower force increases all the time as the elastic foundation parameter increases for a given mass ratio when both the mass effect and the rotary inertia of the tip mass are taken into account. Also, the rotary inertia of the tip mass affect somewhat significantly the critical follower force for a given elastic foundation parameter.

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